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Distribution characteristics of mechanical properties and correlation between the respective properties on S35C carbon steel

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Mechanical properties of tensile strength, σ_B, upper yield stress, σ_{SU}, lower yield stress, σ_{SL}, elongation, δ, area reduction, ϕ, Vickers hardness, H_v, and impact absorbed energy, E, were examined using 50 specimens of S35C carbon steel, which were machined from two bars supplied from the same charged and heat-treated material. Distribution characteristics of these properties are discussed, and the correlation between each pair of them is investigated from a statistical viewpoint. The main conclusions obtained are summarized as follows; distribution characteristics of σ_B, σ_{SL}, δ, ϕ, H_v and E are well approximated by a normal distribution, but those of σ_{SU} are not approximated as well by this type of distribution. In the latter case, a Weibull distribution is preferable to represent the distribution pattern. No significant correlation was observed between each pair of the above mechanical properties. Consequently, individual properties have the inherent distribution characteristics independent of the other properties.

1. Introduction

In reliability analyses of machines and structures, it is necessary to quantitatively know the distribution characteristics of mechanical properties of fabricated parts and structural members [1–3]. These aspects are usually determined through the statistical analysis of the experimental results or practical field data [4]. Mechanical structures are typically composed of numerous parts fabricated from many different metallic materials. The accumulation and reporting of numerical data and the development of a database on the strengths of materials have been approached in the same manner [5, 6]. Moreover, theoretical interpretations were attempted on the distribution pattern of mechanical properties by many researchers [7–11].

The above recent works are focused on the distribution characteristics of the individual properties such as tensile strength, yield stress, hardness, toughness, fatigue strength and so on. However, systematic observations were not performed on the statistical aspects of the serial mechanical properties of a provided material. Of course, each of the respective strengths has a scatter. Is this distribution pattern inherent to the individual strength component? Is there any mutual correlation among the serial properties?

Let us consider a long rolled steel bar. If the strength distributes along the location of the bar and a correlation is found among the respective strengths, a portion of the bar giving high tensile strength provides the corresponding values of the other properties such as hardness and impact absorbed energy. On the contrary, if the respective strengths are statistically independent of each other, the tensile strength of a definite portion provides no information on the other properties of the same portion. Which point of view is predominant in the actual metallic materials? This is the fundamental motivation of the present study.

In this work, statistical aspects of the serial mechanical properties were examined by selecting a typical carbon steel for machine structural use (JIS:S35C). From two long bars (22 mm diameter × 7 m), 50 short segments were cut, after which a set of three specimens for tensile, hardness and impact tests were machined from each segment. Thus the serial strengths from the same portion of the material were obtained. Repeating these tests 50 times, distribution characteristics of the respective strengths were individually observed and mutual correlations between them are discussed.

2. Material and experimental procedure

The material used in the present work is a carbon steel for machine structural use (JIS:S35C) supplied as straight rolled bars with a diameter of 22 mm and a length of 7 m. Two bars of this material were directly heat treated at 850 °C for 1 h in a large furnace after confirming a uniform distribution of the temperature. Because the tensile, hardness and impact specimens would be cut from the same portion due to the main aim in this study, the respective specimens were prepared as follows.

The heat-treated bars were cut into segments and sequentially numbered as shown in Fig. 1a, after which tensile specimens were machined into the shape

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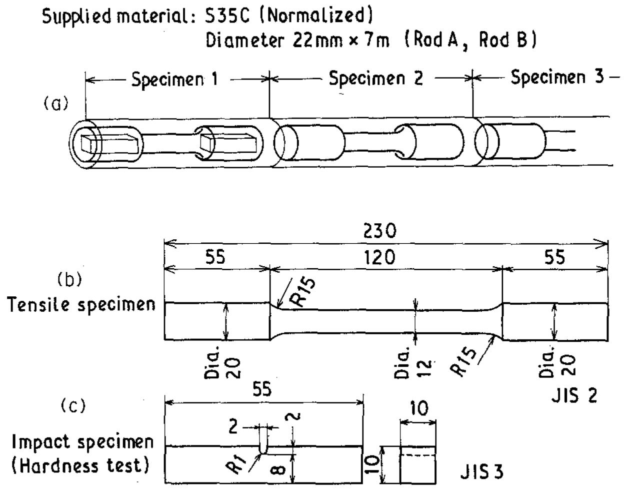

Figure 1 Shapes and dimensions (mm) of specimens.

shown in Fig. 1b (JIS:Specimen 2). One of two bars provided 24 specimens and the other 26 specimens. The former is hereafter referred to as "Rod A", and the latter as "Rod B". When the experimental results from both rods are combined, another notation "Rods A + B" is used for the sake of convenience. Impact specimens were machined from the chucking portions of the tensile specimens as illustrated in Fig. 1a after tensile tests were accomplished. The stress in the chucking portion does not exceed the yield stress during the entire period of the tensile test, and, therefore, work hardening does not take place in this part of the tensile test specimen. The shape and dimensions of this impact specimen, of a Charpy-type geometry, are indicated in Fig. 1c (JIS:Specimen 3). Thus we have two impact specimens from each of the tensile specimens. Vickers hardness was measured on the surface of the impact specimen so that the hardness of the inner portion of the rolled bar, approximately corresponding to the surface of tensile specimen, could be obtained. Measurements of the hardness were repeated ten times on each one of the impact specimens. Consequently, in each segment of Specimens 1, 2, etc., in Fig. 1a, we can obtain one specimen for tensile testing and two specimens for impact testing which also provide the specimens for hardness testing. Impact absorbed energy and hardness are thus provided by the average values experimentally obtained on a pair of specimens. Thus we can obtain the tensile properties (tensile strength, σ_B, upper and lower yield stresses, σ_{SU} and σ_{SL}, elongation, δ, area reduction, ϕ), the Vickers hardness, H_v, and the impact absorbed energy, E, on the same portion of the long-rolled bar.

Table I indicates the chemical compositions of the test material of both Rods A and B. In order to investigate the fluctuation of the compositions, analysis was completed on three different locations in each bar. Location 1 implies one end of the bar, location 2 the central part, and location 3 the opposite end. Fluctuation of the chemical compositions is negligible between Rods A and B, and their fluctuation depending the location along the rolled bar is also negligibly small.

3. Results and discussions

By means of the procedures mentioned previously, tensile property, Vickers hardness and impact absorbed energy in Charpy type specimens were repeatedly examined on a total of 50 segments (Rod A, 24 segments; Rod B, 26 segments). Distribution characteristics of the mechanical properties were individually investigated for Rod A and Rod B as well as for the combined data of Rods A + B. Furthermore, the significant difference in the distribution characteristics of the respective properties were examined between Rods A and B, and the correlations between any pair of the properties were also investigated from the statistical viewpoint.

3.1. Distribution characteristics of respective mechanical properties

3.1.1. Tensile strength

Distributions of tensile strength, σ_B(MPa), are plotted on normal probability paper in Fig. 2, where individual distributions of Rods A and B are shown on the left-hand side, and the combined distribution on Rods A + B is on the right-hand side. The abscissa is shifted a little to plot the combined data in order to avoid overlapping of the data points. Cumulative frequency of the ordinate, F, is calculated by $F(x_i) = (i - 0.5)/n$, where x_i is the ith datum from the minimum of n total data [4, 12].

The straight lines in Fig. 2 indicate the normal distribution functions determined by the following sample mean, m, and standard deviation, s

$$m = \frac{1}{n}\Sigma x_i \quad (1)$$

$$s^2 = \frac{1}{n-1}\Sigma (x_i - m)^2 \quad (2)$$

Although a slight difference of the standard deviation is found between Rods A and B, the mean values are in good agreement with one another. Equalities of the mean and standard deviation between Rods A and B

TABLE I Chemical compositions (wt %)

Rod	Loc.	C	Si	Mn	P	S	Cu	Cr	Ni
A	1	0.37	0.21	0.73	0.019	0.017	0.005	0.053	0.018
	2	0.37	0.22	0.73	0.020	0.018	0.005	0.053	0.017
	3	0.37	0.21	0.73	0.020	0.018	0.005	0.054	0.018
B	1	0.36	0.21	0.74	0.020	0.017	0.006	0.054	0.019
	2	0.37	0.21	0.73	0.019	0.018	0.005	0.052	0.017
	3	0.37	0.20	0.72	0.018	0.018	0.005	0.052	0.017

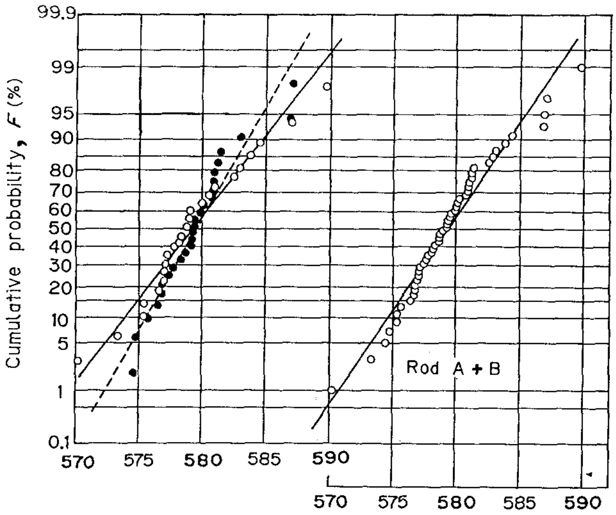

Figure 2 Distributions of tensile strength, σ_B. (○) Rod A, (●) Rod B.

will be discussed later by means of the statistical test technique. Combined data of Rods A + B closely approximate a straight line, as indicated on the right-hand side. Therefore, the distribution of the tensile strength, σ_B, is well represented by the following normal distribution

$$f(x) = \frac{1}{(2\pi)^{1/2}s}\exp\left[-\frac{1}{2}\left(\frac{x-m}{s}\right)^2\right] \qquad (3)$$

A random variable x is used here to designate the strength components. This consists of the tensile strength, σ_B, in this case, and we can set x equal to σ_B.

3.1.2. Yield stress

Distribution characteristics of the upper yield stress, σ_{SU}, are similarly depicted in Fig. 3. Linearity of the plotted data is not as good for the individual Rods A and B, nor on the combined data of Rods A + B. Each of them reveals a slight deviation from the regression line and each group tends to have a convex pattern upwards in this coordinate. Another finding is that the standard deviation of σ_{SU} is relatively larger than that of the tensile strength, σ_B.

It should be noted that the normal distribution is an infinite distribution over the range of $-\infty < x < \infty$. However, mechanical properties such as tensile strength and yield stress cannot have negative values. As a result, a certain discrepancy is introduced into the formulation of the distribution pattern given in Equation 3. The marked fitness of the tensile strength to a normal distribution in Fig. 2 is based on the fact that σ_B has a relatively high mean value and a small standard deviation. In such a case, the probability density in the negative region of x becomes negligibly small. This is the reason why the distribution of σ_B can be well represented by a normal distribution in the form of Equation 3.

In the case of upper yield stress, σ_{SU}, the mean value is less than that of σ_B, but the standard deviation is larger than that of σ_B. Therefore, the above dis-

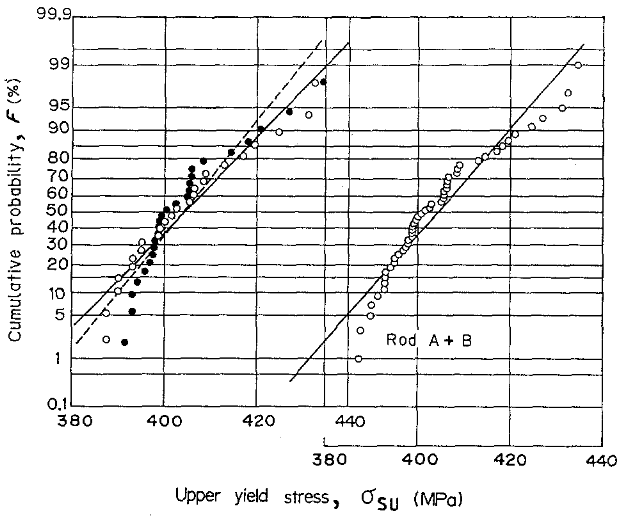

Figure 3 Distributions of upper yield stress, σ_{SU}, (normal probability paper). (○) Rod A, (●) Rod B.

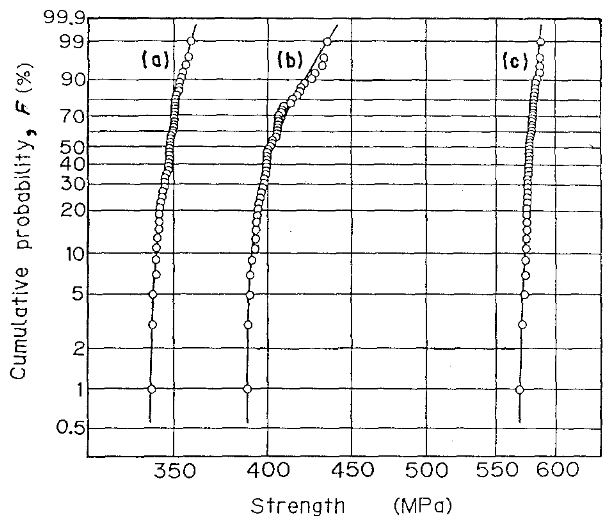

Figure 4. Distributions of (a) σ_{SL}, (b) σ_{SU} and (c) σ_B (Weibull probability paper). (a) $a = 3.30$, $b = 15.1$, $c = 335$. (b) $a = 1.78$, $b = 21.1$, $c = 385$. (c) $a = 4.80$, $b = 15.7$, $c = 565$.

crepancy becomes obvious in the lower region of the distribution. This the reason why the experimental results tend to appear lower than the straight line in the lower tail in Fig. 3. In this case, a distribution function having the lower bound such as a Weibull type or log-normal type is preferred to represent the distribution characteristics.

From this point of view, the same results on Rods A + B are replotted on Weibull probability paper in Fig. 4, in which tensile strength, σ_B, and lower yield stress, σ_{SL}, are also plotted for the sake of comparison. Solid lines passing through the experimental data are a distribution function having the following form

$$F(x) = 1 - \exp\left[-\left(\frac{x-c}{b}\right)^a\right] \qquad (4)$$

where a, b and c are shape, scale and location parameters respectively, and they can be determined by the correlation coefficient method of parameter estimation [13, 14]. These parameters thus obtained are noted in the figure. This type of distribution function is in excellent agreement with the experimental one for the respective strengths of σ_{SL}, σ_{SU} and σ_B. Consequently, the Weibull distribution is preferable to represent the distribution pattern of the upper yield stress, σ_{SU}; however, both Weibull and normal distributions provide a similar goodness of fit to the experimental distribution of the tensile strength, σ_B. Although the figure is not presented here, it is confirmed that the lower yield stress σ_{SL} is governed by a normal distribution. Thus the insufficient goodness of fit to the experimental distribution seems to be particular to the upper yield stress, σ_{SU}, having a relatively low mean value and high standard deviation.

3.1.3. *Elongation*
Elongation is calculated by $\delta = (l - l_0)/l_0$, where l_0 is the gauge length (100 mm) and l is the length after fracture, and the distribution characteristics of this value for Rods A and B are plotted on a normal probability paper in Fig. 5 together with that of combined data of Rods A + B. It is found that distribution characteristics of this elongation are well represented by a normal distribution function both for the individual rods and the combined data. As a result, the probability density function in Equation 3 can be written by equation x to δ. In Fig. 5, mean values of δ on respective rods agree with each other, but a difference of the standard deviation is observed between Rods A and B. The equality of these distributions for both rods are systematically discussed later, following the technique of the statistical test.

3.1.4. *Area reduction*
Reduction of area is calculated by $\phi = (A_o - A)/A_o$, and the distributions are similarly depicted in Fig. 6. The overall trend of the distribution pattern for ϕ seems to be represented by a normal distribution as indicated by the straight lines obtained in Fig. 6, although a slight gap is observed around the value of $\phi = 57.4\%$. The probability density function of ϕ is directly provided by putting $x = \phi$ in Equation 3. For the area reduction ϕ, a slight difference of the mean value is observed between Rods A and B, while no significant difference is seen for the standard deviation.

3.1.5. *Vickers hardness*
Each of tensile specimens provided two samples for hardness tests, and Vickers hardness, H_v, was repeatedly measured 10 times on each of them. It is assumed that the hardness of the individual test segment is given by the average of these 20 data points. Distribution characteristics of Vickers hardness values thus obtained are plotted on normal probability paper in Fig. 7.

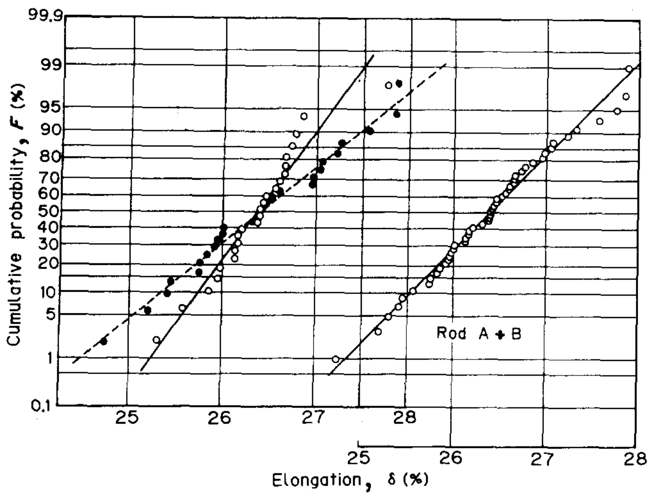

Figure 5 Distributions of elongation, δ. (○) Rod A, (●) Rod B.

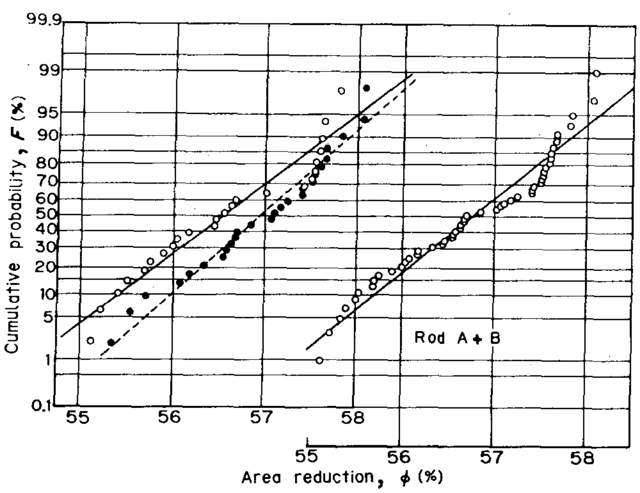

Figure 6 Distributions of area reduction, ϕ. (○) Rod A, (●) Rod B.

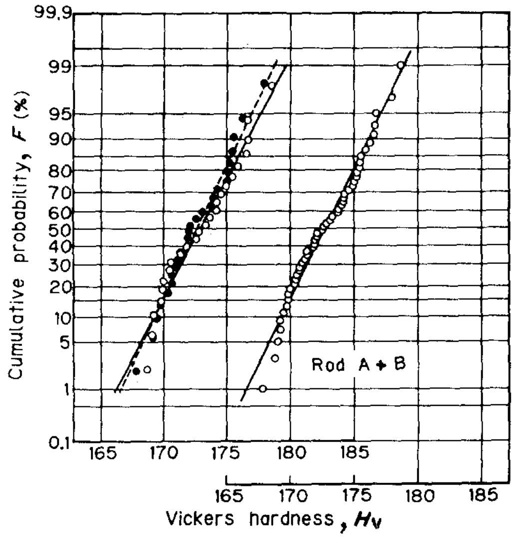

Figure 7 Distributions of Vickers hardness, H_v. (○) Rod A, (●) Rod B.

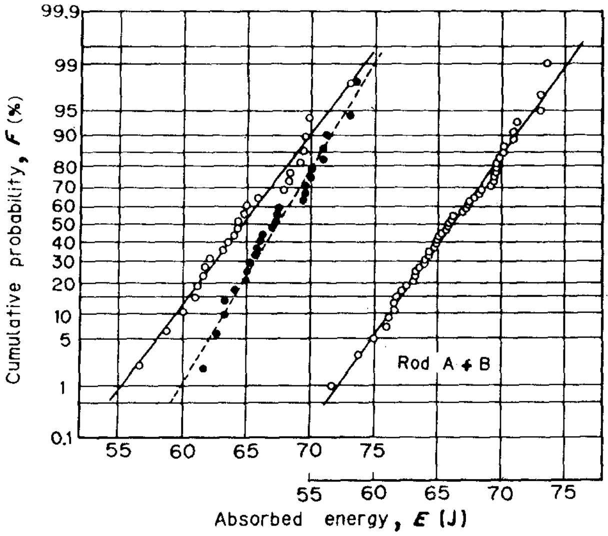

Figure 8 Distributions of absorbed energy, E. (○) Rod A, (●) Rod B.

It is found that the distribution pattern of the hardness is well fitted by normal distributions both for Rods A and B and the combined data of Rods A + B. Moreover, mean values and standard deviations for hardness exhibit no difference between Rods A and B so that the results of Rods A and B are almost overlapping. This result can be attributed to the fact that each data point implies the average of 20 test values. In other words, Fig. 7 indicates the distribution characteristics of the average of 20 data on the hardness. The probability density function of the hardness can be obtained by substituting $x = H_v$ in Equation 3. Nishijima [15] reported that a significant difference in hardness values was observed among many rolled bars even if they were fabricated by similar processing and heat-treatment, contrary to these results. This feature seems to depend on the detailed conditions of the material processing and on the number of measurements. In order to solve this problem, further experimental results should be systematically accumulated in the future.

3.1.6. Charpy impact absorbed energy

Impact tests of a Charpy type were repeatedly performed by using two specimens from every segment depicted in Fig. 1a. Distributions of the absorbed energy in the impact fracture are depicted on normal probability paper in Fig. 8. The distribution pattern of the absorbed energy, E, is well approximated by normal distributions for Rods A and B and the combined data of Rods A + B, as shown by the respective straight lines in Fig. 8. Accordingly, its probability density function is also provided by Equation 3 by substituting $x = E$. For the distribution of this absorbed energy, the mean value and standard deviation are different for Rods A and B. Equality of the distributions for all of the above mechanical properties of Rods A and B are discussed in the next section.

3.2. Statistical tests on equality of two population distributions

In order to examine the equality of the distributions of the mechanical properties of both Rods A and B, equalities of the mean value and standard deviation are confirmed by the following technique of the statistical test.

3.2.1. Equality of standard deviation

Suppose that there are a couple of normal populations $N_1(m_1, s_1)$ and $N_2(m_2, s_2)$ and all of the parameters m_1, s_1, m_2 and s_2 are unknown. Then the postulate that standard deviation (or variance) of these two populations are equal to each other can be statistically tested by means of the F-distribution. The results of such statistical tests for the significance levels of $\alpha = 0.05$ and 0.01 are listed in Table II. "x" indicates that the above postulate should be rejected, while "o" implies that the postulate, $s_1 = s_2$, cannot be rejected. The latter symbol, therefore, indicates that the standard deviations (or variances) of both populations are assured to be equal to each other at the given significance level.

In the case of $\alpha = 0.05$, the above postulate on the equality is rejected for the lower yield strength, σ_{SL}, and the elongation, δ. But if $\alpha = 0.01$, no significant difference between standard deviations of both populations is found for any of the mechanical properties. Consequently, we can state that standard deviations of these mechanical properties are mutually equivalent for Rods A and B at the significance level of $\alpha = 0.01$.

3.2.2. Equality of mean value

The equality of $s_1 = s_2$ was statistically confirmed on every mechanical property developed in the previous

TABLE II Statistical test results for equality of variances

α	σ_B	σ_{SU}	σ_{SL}	δ	ϕ	H_v	E
0.05	○	○	×	×	○	○	○
0.01	○	○	○	○	○	○	○

TABLE III Statistical test results for equality of means

α	σ_B	σ_{SU}	σ_{SL}	δ	ϕ	H_v	E
0.05	○	○	○	○	○	○	×
0.01	○	○	○	○	○	○	○

section. Therefore, we can perform the statistical tests on the equality of the mean values of the respective properties between Rods A and B as follows.

Let us denote a couple of normal populations having the same standard deviation $s_1 = s_2 = s$, as $N_1(m_1, s)$ and $N_2(m_2, s)$. Then, by applying the t-distribution, we can statistically test a postulate that the mean values of both populations are equal to each other, $m_1 = m_2$. Results of such statistical tests at the significance levels of $\alpha = 0.05$ and 0.01 are given in Table III. "x" and "o" indicate whether the postulate $m_1 = m_2$ should be rejected, as used similarly in Table II. In the case of $\alpha = 0.05$, the postulate is rejected only for the impact absorbed energy E. However, if $\alpha = 0.01$, the above postulate is not rejected for any of the mechanical properties. Consequently, at the significance level of $\alpha = 0.01$, we can conclude that mean values of the respective properties are equal to each other for Rods A and B.

The statistical tests discussed above are developed for populations conforming to normal distributions. However, the fitness of the distribution is not good for the upper yield stress, δ_{SL}, as discussed in Section 3.1.2. Therefore, by using a non-parametric technique of the run-test, the statistical test was again performed on the equality of the distributions for Rods A and B. This test results in the population distributions of Rods A and B which had no significant difference at a level of $\alpha = 0.05$.

Thus, it can be stated that the experimental results of the mechanical properties of Rods A and B belong to a common population. In other words, we can combine the data on Rods A and B as a set of experimental results. From this point of view, original distributions of the mechanical properties of the present material should be determined by the combined data of Rods A + B. Such original distributions can provide the fundamental data in the reliability-based design of machines and structures.

3.3. Correlation between respective mechanical properties

Suppose that the tensile strength, σ_B, is high in a certain segment shown in Fig. 1a. Then, are other strength components in the same portion affected by this level of σ_B? This is another interesting question. Fig. 9 shows the correlations between σ_B and the mechanical properties determined in this study. In each diagram, no significant correlation is found between the tensile strength and other properties both for the individual rods (Rods A and B) and for the combined data (Rods A + B), although a weak correlation is observed between σ_B and the lower yield stress, σ_{SL}. Moreover, both symbols (○ and ●) al-

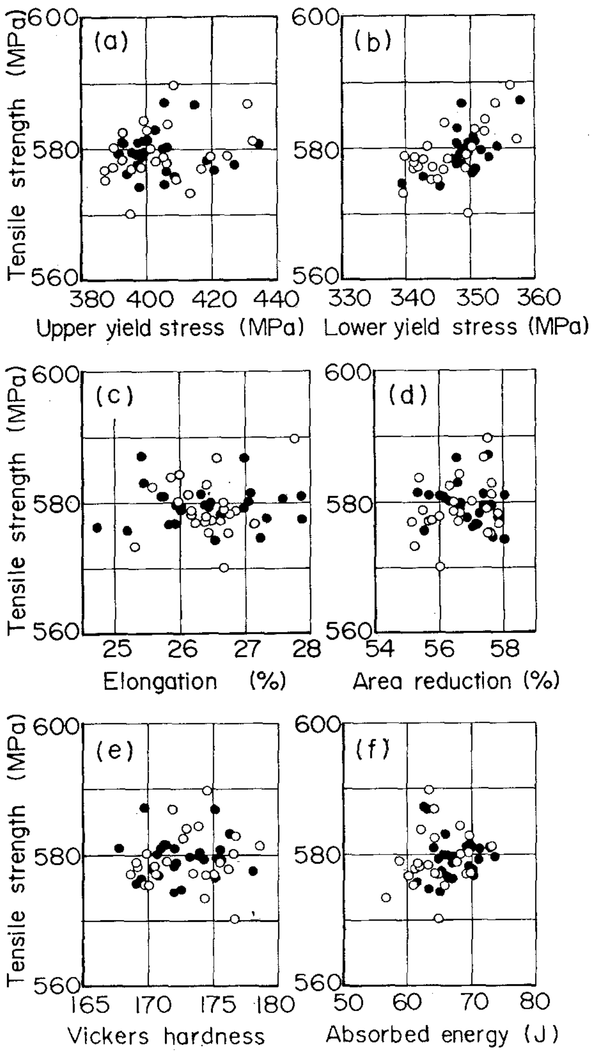

Figure 9 Correlation between tensile strength and other mechanical properties for Rods (○) A and (●) B.

most overlap within the same scattering area. This can be attributed to the fact that both mean value and standard deviation of the respective strength components are equal to each other for Rods A and B.

Attempts can be made to determine whether correlations exist between any of the other mechanical properties obtained in this study. In order to examine the correlation of every combination, correlation coefficients were calculated for all of the possible pairs.

Correlation coefficients thus obtained are listed in Table IV, in which the values calculated for the individual rods (Rods A and B) are presented in parentheses and the values for the combined data (Rods A + B) are indicated underneath the above values in the corresponding column. Six combinations in the first line in Table IV correspond to the respective diagrams in Fig. 9. A weak correlation was recognized in the σ_B–σ_{SL} relationship in Fig. 9b. This result coincides with the fact that the correlation coefficient in this combination also has the highest value of R, equal to 0.58. However, other combinations in this line have low values of R, less than 0.2. From the numerical data in Table IV, it is found that the correlation between each pair of mechanical properties is sufficiently weak, R is less than 0.3, except for the pair of σ_B–σ_{SL} with $R = 0.58$. Consequently, distribution characteristics of

TABLE IV Correlation between each pair of mechanical properties

	Rod	σ_{SU}	σ_{SL}	δ	ϕ	H_v	E
σ_B	A	(0.29)	(0.60)	(0.16)	(0.28)	(0.10)	(0.23)
	B	(0.02)	(0.57)	(0.00)	(− 0.21)	(0.09)	(− 0.03)
	Total	0.18	0.58	0.07	0.09	0.09	0.13
σ_{SU}	A	—	(0.26)	(− 0.07)	(0.13)	(0.16)	(0.04)
	B		(− 0.14)	(0.30)	(− 0.19)	(0.12)	(0.13)
	Total		0.10	0.14	− 0.01	0.14	0.07
σ_{SL}	A		—	(0.19)	(0.43)	(0.36)	(0.30)
	B			(− 0.18)	(− 0.01)	(− 0.05)	(0.11)
	Total			− 0.01	0.30	0.18	0.30
δ	A			—	(0.46)	(0.00)	(− 0.02)
	B				(0.25)	(0.41)	(0.14)
	Total				0.31	0.24	0.07
ϕ	A				—	(− 0.14)	(0.06)
	B					(0.27)	(− 0.10)
	Total					0.03	0.06
H_v	A					—	(0.31)
	B						(− 0.02)
	Total						0.13

the respective strength components as shown in Section 3.1 seem to be independent of the other components, although a weak correlation is observed between the tensile strength and the lower yield stress.

4. Conclusions

Serial mechanical properties obtained from tensile, hardness and impact tests were systematically examined using fifty specimens of S35C carbon steel, which were machined from two bars supplied as the same charged and heat-treated material. Distribution characteristics of these properties were discussed, and the correlation between each pair was also investigated from the statistical viewpoint. The main conclusions obtained in this study are summarized below.

1. Distribution characteristics of tensile strength, σ_B, lower yield stress, σ_{SL}, elongation, δ, area reduction, ϕ, Vickers hardness, H_v, and Charpy impact absorbed energy, E, are well represented by normal distributions. However, for the upper yield stress, σ_{SU}, the fitness of the normal distribution is not good and three-parameter Weibull distribution is prefered to represent its distribution pattern.

2. Specimens were cut out from two supplied bars (Rods A and B). Based on the statistical test of the equality for mean and standard deviation between these two different bars, no significant difference was found for all of the mechanical properties at the significance level of $\alpha = 0.01$. Distribution characteristics determined from the combined data (Rods A + B) can provide the fundamental data required for the reliability-based design of machines and structures.

3. For this test material, no significant correlation was found between any pair of mechanical properties. Therefore, it is concluded that each one of the respective properties has the inherent distribution characteristics independent of the other types of strength components. However, a weak correlation having the correlation coefficient of $R = 0.58$ was observed between the tensile strength, σ_B, and the lower yield stress, σ_{SL}.

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